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Thermomechanical Effects in Cholesteric Liquid Crystals

G. S. Ranganath [a]

[a] Raman Research Institute, Bangalore, 560080, India

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THERMOMECHANICAL EFFECTS IN CHOLESTERIC LIQUID CRYSTALS

G.S.RANGANATH
Raman Research Institute, Bangalore 560080, India

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ABSTRACT. Thermomechanical effects in cholesteric liquid crystals are considered in the coarse-grained approximation of Leslie's theory. In certain situations one finds a close analogy with superfluid hydrodynamics. Effects of permeation have also been discussed.

THEORY

Leslie[1] has developed a continuum theory of cholesteric liquid crystals. This theory can be simplified considerably in the coarse-grained approximation. Here one assumes all variables that describe distortions and motions to be varying slowly and smoothly over many pitches. Further one ignores director inertia and all higher order terms and averages out all the high frequency components. Then Leslie's equations become:

$$\frac{1}{\rho}\frac{\partial\rho}{\partial t} + v_{i,i} = 0$$

$$\rho\dot{v}_i = -P,_i + \delta_{i3}\, g + t'_{ij,j}$$

$$\dot{u} - v_3 = \left(\frac{-1}{\lambda_1 q^2}\right) g + \left(\frac{-\lambda_3}{\lambda_1 q}\right) T,_3$$

$$\dot{Q} = K_{\parallel} T_{,33} + K_{\perp}(T_{,11} + T_{,22}) + qK_3(\dot{u} - v_3)_{,3} .$$

Here q is the wave vector of the unperturbed cholesteric structure, uq represents changes in the director orientation in its own plane in each cholesteric layer (phase fluctuations). g is the hydrostatic force parallel to the twist axis arising from static distortions in the cholesteric. The hydrodynamic stress t'_{ij} also gets greatly simplified in the coarse-grained approximation. The thermomechanical coefficients λ_3 and K_3 become equal in magnitude and of opposite signs if we invoke Onsager's principle.[2]

It is important to remark that the above equations are similar in content and structure to those obtained by Lubensky[3] and Martin, Parodi and Pershan,[4] on general thermodynamical considerations. In this model t'_{ij} is strictly symmetric.

RESULTS

It is easy to verify that the above equations admit solutions describing Helfrich's permeation[5,6] and Lehmann's rotation phenomenon.[7,8] Though permeation is rather clearly established, Lehmann's rotation appears to have not been reproduced after Lehmann's discovery. In view of this we look for other consequences of thermomechanical coupling which appear not to have been noticed.

(a) No permeation ($\dot{u} = v_3$)

(i) A steady flow is allowed (by the above equations) along the z-axis. In the non-dissipative limit for such a flow we find

$$P_{,3} = -\lambda_3 q T_{,3} \quad \text{or} \quad \frac{\partial P}{\partial T} = -\lambda_3 q \quad .$$

For negative λ_3 (q being positive for a right handed structure) this is the analogue of the London equation for a superfluid[9,10] which explains the fountain effect. This flow carries heat with it resulting in an extra thermal conductivity which can be computed as in superfluids by including the small viscous terms in t'_{ij}. This extra conductivity turns out to be

$$K_S^e = \left(-\frac{\lambda_3 q}{\eta} \beta\right) \rho S T \quad .$$

Again as in superfluids β depends on the geometry of the channel. Note the linear dependence of K_S^e on T. This is the analogue of the thermal superconductivity in superfluids.[10]

The above remarks pertain to negative λ_3. However λ_3 can also be positive. In such a case there can be a decrease in pressure in the high temperature region and K_S^e can be negative.

(ii) We next consider wave propagation in cholesterics. Apart from the familiar longitudinal density wave there is in fact an interesting mode akin to second sound in superfluids: a transverse wave travelling along directions <u>not exactly</u> coinciding with either the twist axis or a normal to it. Its velocity for a direction θ with respect to the z axis is

$$c = (K_{22}/\rho)^{\frac{1}{2}} q \sin\theta \cos\theta$$

As in superfluids this wave has phase fluctuations. It also has temperature fluctuations accompanying it provided

$\lambda_3 > 0$. Then the ratio of the amplitude of the temperature wave to that of the phase wave is

$$f = -\frac{K_{22}q}{\lambda_3}$$

However, for $\lambda_3 < 0$ the temperature wave is non-propagating It must be remarked that Lubensky[3] was the first to indicate the possibility of second sound as phase fluctuations in cholesterics. But he and others do not appear to have noticed that it has accompanying temperature fluctuations for $\lambda_3 > 0$.

(b) <u>With permeation (or blocked layers)</u> ($\dot{u} = 0$)

(i) In the absence of a pressure gradient, for a temperature gradient along z,

$$v_3 = \left(\frac{\lambda_3}{\lambda_1 q}\right) T_{,3}$$

with no distortions in the cholesteric structure. This phenomenon in which temperature gradient induces a permeation flow may be termed as thermal permeation. (This is the opposite of what Prost[2] has considered. He indicates a generation of temperature gradient under permeation brought about by an imposed pressure gradient.) Interestingly this flow can be along or opposite to the imposed temperature gradient. The heat carried by this process results in an extra thermal conductivity

$$K_P^e = \left(\frac{\lambda_3}{\lambda_1 q}\right) \rho\ S\ T$$

Again this can be positive or negative.

(ii) Dubois-Violette and de Gennes[11] have shown that gravity can sustain, under an imposed temperature fluctuation a new type of convection controlled primarily by permeation. This convection is in many ways different from what one finds in normal fluids. The wave vector is very small and flow patterns very different. Interestingly, due to thermomechanical effect alone one finds a similar instability. The flow velocities are somewhat different. For example under an imposed temperature distribution $T = A \sinh \bar{k}z \cos kx$ ($\bar{k} = k(K_{\parallel}/K_{\perp})^{\frac{1}{2}}$ and $k < \pi/d$) we get

$$v_1 = - A \left(\frac{\lambda_3}{\lambda_1}\right) \left(\frac{\bar{k}^2}{kq}\right) \sinh \bar{k}z \sin kx$$

$$v_2 = 0$$

$$v_3 = + A \left(\frac{\lambda_3}{\lambda_1}\right) \left(\frac{\bar{k}}{q}\right) \cosh \bar{k}z \cos kx \quad .$$

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REFERENCES

1 F.M.Leslie, Mol.Cryst.Liq.Cryst. 7, 407 (1969).

2 J.Prost, Solid State Comm. 11, 183 (1972).

3 T.C.Lubensky, Phys. Rev. A6, 452 (1972).

4 P.C.Martin, O.Parodi and P.S.Pershan, Phys. Rev. A6, 2401 (1972).

5 W.Helfrich, Phys. Rev. Lett. 23, 372 (1969).

6 U.D.Kini, G.S.Ranganath and S.Chandrasekhar, Pramana 5, 105 (1975).

7 O.Lehmann, Ann.Phy. (Leipzig), 2, 649 (1900).

8 F.M.Leslie, Proc. Roy. Soc. A307, 359 (1968).

9 L.D.Landau and E.M.Lifshitz, Fluid Mechanics, Pergamon (1966).

10 S.J.Putterman, Superfluid Hydrodynamics, North-Holland (1974).

11 E.Dubois-Violette and P.G. de Gennes, J.Phys.(Paris), Supplement 36, C1-293 (1975).